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Book Review: *Introduction to Nonlinear Science*

Introduction to Nonlinear Science. G. Nicolis, Cambridge University Press, Cambridge, England, 1995.

The first noticeable feature of this book is the cover, which features Peter Bruegel's painting *The Tower of Babel.* The appearance of this work of art on the cover of a scientific text is explainable as follows: in the early days of modern science natural philosophy was considered a subject worthy of research. This type of research was quite general, being exemplified by unified approaches to this subject by da Vinci or Newton. Afterward, somewhat more self-assured scientists set out to penetrate the secrets of the universe. Because of this, an angry God distributed among them a great number of scientific languages with the result that applied mathematicians, physicists, chemists, biologists, engineers, geophysicists, ecologists, etc., are no longer able to understand one another. However, scientists have no intention of surrendering in this unequal fight, and are now trying to find a common language. If they succeed, they will have erected a new scientific Tower of Babel of unprecedented height. Nonlinear science would necessarily be an important part of such a language.

This book consists of material for an undergraduate course in physics and chemistry that has been taught by author at the University of Brussels. It contains seven chapters, the first two of which discuss several standard models of nonlinearity. Chapters 3–5 develop the theory of dynamic systems with finite degrees of freedom and Chapter 6 is dedicated to the analysis of spatially extended systems. The final chapter describes and develops the theory of chaotic dynamics.

Chapter 1 contains a general description of phenomena in which nonlinearity plays an important role (e.g. a pendulum with a rotating point of support, thermal convection, chemical reactions, as well as a variety of biological phenomena). Chapter 2 describes quantitative approaches to the models in Chapter 1. Some of these are presented in no more than telegrammatic form. Chapter 3 discusses concepts needed to describe geometric representations of mechanical systems. Some of these include the notion of phase space, invariant manifolds for systems of different dimensions,

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fractal manifolds, conservative and dissipative systems, attractors, and stability analysis. Chapter 4 discusses linear stability analysis near fixed points for multidimensional systems. Chapter 5 presents a unified bifurcation analysis of nonlinear systems, classifying transcritical, pitchfork, saddle-node, Hopf, and cascading bifurcations. Analogies are also drawn to the theory of equilibrium phase transitions. Chapter 6 describes a generalization of bifurcation to the analysis of spatially distributed systems. Linear stability analyses are carried out on equations that describe the Benárd problem and for reaction-diffusion systems. A bifurcation analysis beyond the instability point is carried out for the Benárd problem in the nonlinear Boussinesq approximation (small aspect ratio) and for the reactive diffusion described by the complex Landau–Ginzburg equation (large spatial extent). The use of Poincaré maps is illustrated in the context of deterministic chaos in Chapter 7. The analysis starts from a consideration of one-dimensional maps (logistic, circle, and intermittent). Global properties are then considered using the ideas embodied in coarse-graining, symbolic dynamics, and self-similarity. The probability density is used to analyze such important properties of established chaos as the sensitivity of system dynamics to initial conditions, ergodicity, and dynamic properties. Relations to information theory and random processes are emphasized. A short description of spatiotemporal chaos completes this chapter. Each chapter ends with a set of problems.

Not all parts of this book are equally valuable. The treatment of stochastic aspects of chaos in Chapter 7 is quite original, and the bifurcation analysis in Chapters 5 and 6 is pedagogically successful. At the same time, Chapter 1, while having no formulas discusses many important concepts. If "symmetry breaking" and "bifurcation" are intuitively clear, the hardly understandable ideas of "periodic, quasi-periodic, and chaotic regimes" or "deterministic chaos and random noise" may leave an unready reader in a state of prostration, causing him or her to turn to some other book at a very early stage. This would be a real pity. In general, I belong to the school which supposes that clearly expressed mathematics can serve to simplify the understanding of scientific problems.

My general impression is that the author made an attempt to squeeze too many topics into a book of a restricted size. Apart from this, this book can be recommended to graduate students and researchers for independent study as well as being a basis for an advanced undergraduate course. The excellent typography of Cambridge University Press should be specially commended.

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